

# RUMUS-RUMUS TRIGONOMETRI

## D. Rumus Jumlah dan Selisih Sinus dan Kosinus

Rumus jumlah dan selisih sinus dan kosinus merupakan bentuk manipulasi dari rumus hasil kali sinus dan kosinus yang telah dibahas sebelumnya.

Proses selengkapnya adalah sebagai berikut :

Misalkan  $A = \alpha + \beta$  dan  $B = \alpha - \beta$ , maka

$$\begin{array}{r} A = \alpha + \beta \\ B = \alpha - \beta \\ \hline A + B = 2\alpha \\ \text{Jadi } \alpha = \frac{1}{2}(A + B) \end{array} \quad + \quad \begin{array}{r} A = \alpha + \beta \\ B = \alpha - \beta \\ \hline A - B = 2\beta \\ \text{Jadi } \beta = \frac{1}{2}(A - B) \end{array} \quad -$$

Sehingga diperoleh rumus

$$2.\sin \alpha.\cos \beta = \sin (\alpha + \beta) + \sin (\alpha - \beta)$$

$$2.\sin \frac{1}{2}(A + B). \cos \frac{1}{2}(A - B) = \sin A + \sin B$$

Jadi

$$\sin A + \sin B = 2.\sin \frac{1}{2}(A + B). \cos \frac{1}{2}(A - B)$$

$$2.\cos \alpha.\sin \beta = \sin (\alpha + \beta) - \sin (\alpha - \beta)$$

$$2.\cos \frac{1}{2}(A + B). \sin \frac{1}{2}(A - B) = \sin A - \sin B$$

Jadi

$$\sin A - \sin B = 2.\cos \frac{1}{2}(A + B). \sin \frac{1}{2}(A - B)$$

$$2.\cos \alpha.\cos \beta = \cos (\alpha + \beta) + \cos (\alpha - \beta)$$

$$2.\cos \frac{1}{2}(A + B). \cos \frac{1}{2}(A - B) = \cos A + \cos B$$

Jadi

$$\cos A + \cos B = 2.\cos \frac{1}{2}(A + B). \cos \frac{1}{2}(A - B)$$

$$-2 \sin \alpha \sin \beta = \cos(\alpha + \beta) - \cos(\alpha - \beta)$$

$$-2 \sin \frac{1}{2}(A + B) \sin \frac{1}{2}(A - B) = \cos A - \cos B$$

Jadi

$$\cos A - \cos B = -2 \sin \frac{1}{2}(A + B) \sin \frac{1}{2}(A - B)$$

Untuk lebih jelasnya, pelajari contoh soal berikut ini:

04. Tentukanlah nilai dari :

(a)  $\sin 75^\circ - \sin 15^\circ$

(b)  $\cos 165^\circ + \cos 75^\circ$

Jawab

$$\begin{aligned} \text{(a) } \sin 75^\circ - \sin 15^\circ &= 2 \cos \frac{1}{2}(75^\circ + 15^\circ) \sin \frac{1}{2}(75^\circ - 15^\circ) \\ &= 2 \cos 45^\circ \sin 30^\circ \\ &= 2 \cdot \left(\frac{1}{2}\sqrt{2}\right) \left(\frac{1}{2}\right) \\ &= \frac{1}{2}\sqrt{2} \end{aligned}$$

$$\begin{aligned} \text{(b) } \cos 165^\circ + \cos 75^\circ &= 2 \cos \frac{1}{2}(165^\circ + 75^\circ) \cos \frac{1}{2}(165^\circ - 75^\circ) \\ &= 2 \cos 120^\circ \sin 45^\circ \\ &= 2 \cdot \left(-\frac{1}{2}\right) \left(\frac{1}{2}\sqrt{2}\right) \\ &= -\frac{1}{2}\sqrt{2} \end{aligned}$$

05. Tentukanlah nilai dari :

(a)  $\cos 195^\circ - \cos 45^\circ + \cos 75^\circ$

(b)  $\sin 105^\circ + \sin 195^\circ - \sin 15^\circ + \sin 75^\circ$

Jawab

$$\begin{aligned} \text{(a) } \cos 195^\circ - \cos 45^\circ + \cos 75^\circ &= \cos 195^\circ + \cos 75^\circ - \cos 45^\circ \\ &= 2 \cos \frac{1}{2}(195^\circ + 75^\circ) \cos \frac{1}{2}(195^\circ - 75^\circ) - \cos 45^\circ \\ &= 2 \cos 135^\circ \cos 60^\circ - \cos 45^\circ \\ &= 2 \left(-\frac{1}{2}\sqrt{2}\right) \left(\frac{1}{2}\right) - \left(\frac{1}{2}\sqrt{2}\right) \\ &= -\frac{1}{2}\sqrt{2} - \frac{1}{2}\sqrt{2} \\ &= -\sqrt{2} \end{aligned}$$

$$\begin{aligned}
(b) \quad & \sin 105^\circ + \sin 195^\circ - \sin 15^\circ + \sin 75^\circ \\
&= \sin 105^\circ - \sin 15^\circ + \sin 195^\circ + \sin 75^\circ \\
&= 2 \cdot \cos \frac{1}{2}(105^\circ + 15^\circ) \cdot \sin \frac{1}{2}(105^\circ - 15^\circ) + 2 \cdot \sin \frac{1}{2}(195^\circ + 75^\circ) \cdot \cos \frac{1}{2}(195^\circ - 75^\circ) \\
&= 2 \cdot \cos 60^\circ \cdot \sin 45^\circ + 2 \cdot \sin 135^\circ \cdot \cos 60^\circ \\
&= 2 \left(\frac{1}{2}\right) \left(\frac{1}{2}\sqrt{2}\right) + 2 \left(\frac{1}{2}\sqrt{2}\right) \left(\frac{1}{2}\right) \\
&= \frac{1}{2}\sqrt{2} + \frac{1}{2}\sqrt{2} \\
&= \sqrt{2}
\end{aligned}$$

06. Buktikanlah bahwa :

$$(a) \quad \cos 7x + \cos x + \cos 5x + \cos 3x = 4 \cdot \cos 4x \cdot \cos 2x \cdot \cos x$$

$$(b) \quad \sin 10x + \sin 8x + \sin 4x + \sin 2x = 4 \cdot \cos 3x \cdot \sin 6x \cdot \cos x$$

Jawab

$$(a) \quad \text{Ruas Kiri} = \cos 7x + \cos x + \cos 5x + \cos 3x$$

$$= 2 \cdot \cos \frac{1}{2}(7x + x) \cdot \cos \frac{1}{2}(7x - x) + 2 \cdot \cos \frac{1}{2}(5x + 3x) \cdot \cos \frac{1}{2}(5x - 3x)$$

$$= 2 \cdot \cos 4x \cdot \cos 3x + 2 \cdot \cos 4x \cdot \cos x$$

$$= 2 \cdot \cos 4x (\cos 3x + \cos x)$$

$$= 2 \cdot \cos 4x \cdot 2 \cdot \cos \frac{1}{2}(3x + x) \cdot \cos \frac{1}{2}(3x - x)$$

$$= 4 \cdot \cos 4x \cdot \cos 2x \cdot \cos x$$

$$= \text{ruas kanan}$$

$$(b) \quad \text{Ruas Kiri} = \sin 10x + \sin 8x + \sin 4x + \sin 2x$$

$$= 2 \cdot \sin \frac{1}{2}(10x + 8x) \cdot \cos \frac{1}{2}(10x - 8x) + 2 \cdot \sin \frac{1}{2}(4x + 2x) \cdot \cos \frac{1}{2}(4x - 2x)$$

$$= 2 \cdot \sin 9x \cdot \cos x + 2 \cdot \sin 3x \cdot \cos x$$

$$= 2 \cdot \cos x (\sin 9x + \sin 3x)$$

$$= 2 \cdot \cos x \cdot 2 \cdot \sin \frac{1}{2}(9x + 3x) \cdot \cos \frac{1}{2}(9x - 3x)$$

$$= 4 \cdot \cos x \cdot \sin 6x \cdot \cos 3x$$

$$= 4 \cdot \cos 3x \cdot \sin 6x \cdot \cos x$$

$$= \text{ruas kanan}$$