

# RUMUS-RUMUS TRIGONOMETRI

## C. Rumus Hasil Kali Sinus dan Kosinus

Rumus hasil kali sinus dan kosinus merupakan pengembangan dari rumus jumlah dan selisih dua sudut. Yakni sebagai berikut :

$$\sin(\alpha + \beta) = \sin\alpha \cdot \cos\beta + \cos\alpha \cdot \sin\beta$$

$$\sin(\alpha - \beta) = \sin\alpha \cdot \cos\beta - \cos\alpha \cdot \sin\beta$$

$$\sin(\alpha + \beta) + \sin(\alpha - \beta) = \frac{2 \cdot \sin\alpha \cdot \cos\beta + 0}{+}$$

Jadi

$$2 \cdot \sin\alpha \cdot \cos\beta = \sin(\alpha + \beta) + \sin(\alpha - \beta)$$

$$\sin(\alpha + \beta) = \sin\alpha \cdot \cos\beta + \cos\alpha \cdot \sin\beta$$

$$\sin(\alpha - \beta) = \sin\alpha \cdot \cos\beta - \cos\alpha \cdot \sin\beta$$

$$\sin(\alpha + \beta) - \sin(\alpha - \beta) = \frac{0}{+} + \frac{2 \cdot \cos\alpha \cdot \sin\beta}{-}$$

Jadi

$$2 \cdot \cos\alpha \cdot \sin\beta = \sin(\alpha + \beta) - \sin(\alpha - \beta)$$

$$\cos(\alpha + \beta) = \cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta$$

$$\cos(\alpha - \beta) = \cos\alpha \cdot \cos\beta + \sin\alpha \cdot \sin\beta$$

$$\cos(\alpha + \beta) + \cos(\alpha - \beta) = \frac{2 \cdot \cos\alpha \cdot \cos\beta + 0}{+}$$

Jadi

$$2 \cdot \cos\alpha \cdot \cos\beta = \cos(\alpha + \beta) + \cos(\alpha - \beta)$$

$$\cos(\alpha + \beta) = \cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta$$

$$\cos(\alpha - \beta) = \cos\alpha \cdot \cos\beta + \sin\alpha \cdot \sin\beta$$

$$\cos(\alpha + \beta) - \cos(\alpha - \beta) = \frac{0}{-} - \frac{2 \cdot \sin\alpha \cdot \sin\beta}{-}$$

Jadi

$$-2 \cdot \sin\alpha \cdot \sin\beta = \cos(\alpha + \beta) - \cos(\alpha - \beta)$$

Untuk lebih jelasnya, pelajariilah contoh soal berikut ini:

01. Tentukanlah nilai dari :

(a)  $8.\cos 45^{\circ}.\cos 15^{\circ} + 8.\cos 135^{\circ}.\sin 15^{\circ}$

(b)  $2.\sin 7,5^{\circ} [\cos 52,5^{\circ} + \sin 322,5^{\circ}]$

Jawab

(a)  $8.\cos 45^{\circ}.\cos 15^{\circ} + 8.\cos 135^{\circ}.\sin 15^{\circ}$

$$= 4[\cos(45^{\circ} + 15^{\circ}) + \cos(45^{\circ} - 15^{\circ}) + \sin(135^{\circ} + 15^{\circ}) - \sin(135^{\circ} - 15^{\circ})]$$

$$= 4[\cos 60^{\circ} + \cos 30^{\circ} + \sin 150^{\circ} - \sin 120^{\circ}]$$

$$= 4\left[\frac{1}{2} + \frac{1}{2}\sqrt{3} + \frac{1}{2} - \frac{1}{2}\sqrt{3}\right]$$

$$= 4$$

(b)  $2.\sin 7,5^{\circ} [\cos 52,5^{\circ} + \sin 322,5^{\circ}]$

$$= 2.\cos 52,5^{\circ}.\sin 7,5^{\circ} + 2.\sin 322,5^{\circ}.\sin 7,5^{\circ}$$

$$= 2.\cos 52,5^{\circ}.\sin 7,5^{\circ} - (-2.\sin 322,5^{\circ}.\sin 7,5^{\circ})$$

$$= \sin(52,5^{\circ} + 7,5^{\circ}) - \sin(52,5^{\circ} - 7,5^{\circ}) - \{\cos(322,5^{\circ} + 7,5^{\circ}) - \cos(322,5^{\circ} - 7,5^{\circ})\}$$

$$= \sin 60^{\circ} - \sin 45^{\circ} - \cos 330^{\circ} + \cos 315^{\circ}$$

$$= \frac{1}{2}\sqrt{3} - \frac{1}{2}\sqrt{2} - \frac{1}{2}\sqrt{3} + \frac{1}{2}\sqrt{2}$$

$$= 0$$

02. Buktikanlah bahwa  $4.\cos x.\cos 2x.\sin 3x = \sin 2x + \sin 4x + \sin 6x$

Jawab

Ruas kiri =  $4.\cos x.\cos 2x.\sin 3x$

$$= 2(2.\cos x.\cos 2x).\sin 3x$$

$$= 2(\cos(x + 2x) + \cos(x - 2x))\sin 3x$$

$$= 2.\cos 3x.\sin 3x + 2.\cos(-x).\sin 3x$$

$$= 2.\cos 3x.\sin 3x + 2.\cos x.\sin 3x$$

$$= \sin(3x + 3x) - \sin(3x - 3x) + \sin(x + 3x) - \sin(x - 3x)$$

$$= \sin 6x - \sin 0 + \sin 4x - \sin(-2x)$$

$$= \sin 6x - 0 + \sin 4x + \sin 2x$$

$$= \sin 2x + \sin 4x + \sin 6x$$

$$= \text{ruas kanan}$$

03. Buktikanlah bahwa  $2.\sin(135^\circ + a).\cos(45^\circ - a) = \cos 2a$

Jawab

$$\begin{aligned}\text{Ruas kiri} &= 2.\sin(135^\circ + a).\cos(45^\circ - a) \\ &= \sin([135^\circ + a] + [45^\circ - a]) + \sin([135^\circ + a] - [45^\circ - a]) \\ &= \sin 180^\circ + \sin(90^\circ + 2a) \\ &= 0 + \sin 90^\circ.\cos 2a + \cos 90^\circ.\sin 2a \\ &= (1)\cos 2a + (0).\sin 2a \\ &= \cos 2a \\ &= \text{ruas kanan}\end{aligned}$$