

BARISAN DAN DERET

F. Notasi Sigma

Notasi sigma merupakan bentuk penulisan dari penjumlahan suku-suku $U(1) + U(2) + U(3) + U(4) + \dots + U(n)$, dimana suku-suku tersebut diatur menurut pola tertentu.

Sehingga bentuk umum dari notasi sigma adalah :

$$\sum_{n=p}^q U(n) = U(p) + U(p+1) + U(p+2) + U(p+3) + \dots + U(q)$$

Dimana :
p : Batas Bawah
q : Batas Atas
U(n) : Suku ke-n

Sebagai contoh

01. Uraikanlah bentuk setiap notasi berikut ini

(a) $\sum_{n=2}^{12} (3n+4)$

(b) $\sum_{n=3}^{12} (-1)^n (2)^{n-6}$

Jawab

(a) $\sum_{n=2}^{12} (3n+4) = [3(2)+4] + [3(3)+4] + [3(4)+4] + [3(5)+4] + \dots + [3(12)+4]$
 $= 10 + 13 + 16 + 19 + 22 + \dots + 40$

(b) $\sum_{n=3}^{12} (-1)^n (2)^{n-6} = (-1)^3 (2)^{3-6} + (-1)^4 (2)^{4-6} + (-1)^5 (2)^{5-6} + \dots + (-1)^{12} (2)^{12-6}$
 $= (-1)(2)^{-3} + (2)^{-2} + (-1)(2)^{-1} + (2)^0 + \dots + (2)^6$
 $= -1/8 + 1/4 - 1/2 + 1 + \dots + 64$

02. Ubahlah bentuk uraian berikut ini menjadi notasi sigma dengan batas bawah 3

(a) $5 + 9 + 13 + 17 + \dots + 53$

(b) $42 + 37 + 32 + 27 + \dots - 8$

(c) $2 + 4 + 8 + 16 + \dots + 128$

Jawab

(a) $5 + 9 + 13 + 17 + \dots + 53 = \sum_{n=3}^p (an + b)$

$$5 + 9 + 13 + 17 + \dots + 53 = \sum_{n=3}^p (4n - 7) \quad 4p - 7 = 53$$

$$4p = 60$$

$$p = 15$$

$$5 + 9 + 13 + 17 + \dots + 53 = \sum_{n=3}^{15} (4n - 7)$$

$$(b) 42 + 37 + 32 + 27 + \dots - 8 = \sum_{n=3}^p (an + b)$$

$$42 + 37 + 32 + 27 + \dots - 8 = \sum_{n=3}^p (-5n + 57) \quad -5p + 57 = -8$$

$$-5p = -65$$

$$p = 13$$

$$42 + 37 + 32 + 27 + \dots - 8 = \sum_{n=3}^{13} (57 - 5n)$$

$$(c) 2 + 4 + 8 + 16 + \dots + 128 = \sum_{n=3}^p a^{n-b}$$

$$2 + 4 + 8 + 16 + \dots + 128 = \sum_{n=3}^p 2^{n-2} \quad 2^{p-2} = 128$$

$$2^{p-2} = 2^7$$

$$p - 2 = 7$$

$$p = 9$$

$$81 + 27 + 9 + 3 + \dots + 1/27 = \sum_{n=3}^9 2^{n-2}$$

03. Jika diketahui $\sum_{n=3}^p (an^2 + bn) = 6 + 16 + 30 + 48 + \dots + 240$ maka lengkapilah bentuk

notasi sigmanya

Jawab

$$\sum_{n=3}^p (an^2 + bn) = 6 + 16 + 30 + 48 + \dots + 240$$

$$\text{Maka : } a(3)^2 + b(3) = 6 \text{ maka } 9a + 3b = 6 \dots\dots\dots (1)$$

$$a(4)^2 + b(4) = 16 \text{ maka } 16a + 4b = 16 \dots\dots\dots (2)$$

$$\text{Sehingga } \begin{array}{l} 9a + 3b = 6 \quad (4) \\ 16a + 4b = 16 \quad (3) \end{array} \quad \begin{array}{l} 36a + 12b = 24 \\ 48a + 12b = 48 \\ \hline -12a \quad \quad = -24 \\ \hline a = 2 \end{array}$$

$$9a + 3b = 6$$

$$9(2) + 3b = 6$$

$$18 + 3b = 6$$

$$3b = -12$$

$$b = -4$$

Jadi rumus umum suku ke-n adalah $U_n = 2n^2 - 4n$

$$\sum_{n=2}^p (2n^2 - 4n) = 6 + 16 + 30 + 48 + \dots + 240$$

Sehingga $2p^2 - 4p = 240$
 $p^2 - 2p - 120 = 0$
 $(p - 12)(p + 10) = 0$
 $p = 12$

Jadi $\sum_{n=2}^{12} (2n^2 - 4n) = 6 + 16 + 30 + 48 + \dots + 240$

Terdapat beberapa sifat yang berlaku pada notasi sigma, yakni :

- (1) $\sum_{n=p}^q [a(n) \pm b(n)] = \sum_{n=p}^q a(n) \pm \sum_{n=p}^q b(n)$
- (2) $\sum_{n=p}^q k \cdot a(n) = k \cdot \sum_{n=p}^q a(n)$
- (3) $\sum_{n=p}^q k = (q - p + 1) \cdot k$
- (4) $\sum_{n=p}^q a(n) = \sum_{n=p+r}^{q+r} a(n-r)$ dan $\sum_{n=p}^q a(n) = \sum_{n=p-r}^{q-r} a(n+r)$
- (5) $\sum_{n=p}^q a(n) = \sum_{n=p}^r a(n) + \sum_{n=r+1}^q a(n)$ dimana $p < r < q$

Sifat-sifat di atas dipakai dalam menyelesaikan beberapa soal, sebagai contoh :

04. Buktikanlah bahwa $\sum_{n=2}^8 (4n-3)^2 = 16 \sum_{n=2}^8 n^2 - 24 \sum_{n=2}^8 n + 63$

Jawab

$$\begin{aligned} \sum_{n=2}^8 (4n-3)^2 &= \sum_{n=2}^8 (16n^2 - 24n + 9) \\ &= \sum_{n=2}^8 16n^2 - \sum_{n=2}^8 24n + \sum_{n=2}^8 9 \\ &= \sum_{n=2}^8 16n^2 - \sum_{n=2}^8 24n + (8 - 2 + 1)9 \\ &= 16 \sum_{n=2}^8 n^2 - 24 \sum_{n=2}^8 n + 63 \end{aligned} \quad \text{(terbukti)}$$

05. Ubahlah bentuk $\sum_{n=5}^9 (n^2 - 4n + 2)$ ke dalam notasi sigma dengan

(a) Batas bawah 3

(b) Batas atas 12

Jawab

$$\begin{aligned} \text{(a)} \sum_{n=5}^9 (n^2 - 4n + 2) &= \sum_{n=5-2}^{9-2} ([n+2]^2 - 4[n+2] + 2) \\ &= \sum_{n=3}^7 (n^2 + 4n + 4 - 4n - 8 + 2) \\ &= \sum_{n=3}^7 (n^2 - 2) \end{aligned}$$

$$\begin{aligned} \text{(b)} \sum_{n=5}^9 (n^2 - 4n + 2) &= \sum_{n=5+3}^{9+3} ([n-3]^2 - 4[n-3] + 2) \\ &= \sum_{n=8}^{12} (n^2 - 6n + 9 - 4n + 12 + 2) \\ &= \sum_{n=8}^{12} (n^2 - 10n + 23) \end{aligned}$$

06. Buktikanlah bahwa $\sum_{n=4}^9 (2n+5)^2 = 4 \sum_{n=7}^{12} n^2 - 4 \sum_{n=7}^{12} n + 6$

Jawab

$$\begin{aligned} \sum_{n=4}^9 (2n+5)^2 &= \sum_{n=4+3}^{9+3} (2[n-3]+5)^2 \\ &= \sum_{n=7}^{12} (2n-1)^2 \\ &= \sum_{n=7}^{12} (4n^2 - 4n + 1) \\ &= \sum_{n=7}^{12} 4n^2 - \sum_{n=7}^{12} 4n + \sum_{n=7}^{12} 1 \\ &= 4 \sum_{n=7}^{12} n^2 - 4 \sum_{n=7}^{12} n + (12 - 7 + 1)1 \\ &= 4 \sum_{n=7}^{12} n^2 - 4 \sum_{n=7}^{12} n + 6 \quad \text{(terbukti)} \end{aligned}$$

07. Hitunglah $\sum_{n=5}^{12} (n^2 - 4n + 5) - \sum_{n=7}^{14} (n^2 - 8n + 9)$

Jawab

$$\begin{aligned} \sum_{n=5}^{12} (n^2 - 4n + 5) - \sum_{n=7}^{14} (n^2 - 8n + 9) &= \sum_{n=5}^{12} (n^2 - 4n + 5) - \sum_{n=7-2}^{14-2} ([n+2]^2 - 8[n+2] + 9) \\ &= \sum_{n=5}^{12} (n^2 - 4n + 5) - \sum_{n=5}^{12} (n^2 + 4n + 4 - 8n - 16 + 9) \\ &= \sum_{n=5}^{12} (n^2 - 4n + 5) - \sum_{n=5}^{12} (n^2 - 4n - 3) \\ &= \sum_{n=5}^{12} [(n^2 - 4n + 5) - (n^2 - 4n - 3)] \\ &= \sum_{n=5}^{12} [8] \\ &= (12 - 5 + 1)8 \\ &= 64 \end{aligned}$$

08. Tentukanlah nilai p dan q jika :

(a) $\sum_{n=3}^{18} (n^2 - 4) = \sum_{n=3}^p (n^2 - 4) + \sum_{n=10}^q (n^2 - 4)$

(b) $\sum_{n=2}^{10} (2n + 5) = \sum_{n=2}^{16} (2n + 5) - \sum_{n=p}^q (2n + 5)$

(c) $\sum_{n=6}^{13} (3n - 1) = \sum_{n=1}^{13} (3n - 1) - \sum_{n=p}^q (3n - 1)$

Jawab

(a) $\sum_{n=3}^{18} (n^2 - 4) = \sum_{n=3}^p (n^2 - 4) + \sum_{n=10}^q (n^2 - 4)$

maka $p = 9$ dan $q = 18$

(b) $\sum_{n=2}^{10} (2n + 5) = \sum_{n=2}^{16} (2n + 5) - \sum_{n=p}^q (2n + 5)$

maka $p = 11$ dan $q = 16$

(c) $\sum_{n=6}^{13} (3n - 1) = \sum_{n=1}^{13} (3n - 1) - \sum_{n=p}^q (3n - 1)$

maka $p = 1$ dan $q = 5$

06. Hitunglah $\sum_{n=8}^{12} (4n-4) - \sum_{n=5}^{16} (4n-2) + \sum_{n=13}^{19} (4n-4)$

Jawab

$$\begin{aligned}
 & \sum_{n=8}^{12} (4n-4) - \sum_{n=5}^{16} (4n-2) + \sum_{n=13}^{19} (4n-4) \\
 = & \sum_{n=8}^{12} (4n-4) + \sum_{n=13}^{19} (4n-4) - \sum_{n=5}^{16} (4n-2) \\
 = & \sum_{n=8}^{19} (4n-4) - \sum_{n=5}^{16} (4n-2) \\
 = & \sum_{n=8-3}^{19-3} (4[n+3]-4) - \sum_{n=5}^{16} (4n-2) \\
 = & \sum_{n=5}^{16} (4n+8) - \sum_{n=5}^{16} (4n-2) \\
 = & \sum_{n=5}^{16} [(4n+8) - (4n-2)] \\
 = & \sum_{n=5}^{16} [10] \\
 = & (16-5+1)10 \\
 = & 120
 \end{aligned}$$