

# LIMIT FUNGSI ALJABAR

## B. Teorema Limit

Untuk  $a$ ,  $c$  dan  $n$  adalah bilangan real serta  $f(x)$  dan  $g(x)$  adalah fungsi yang terdefinisi pada real maka berlaku teorema limit:

$$(1) \lim_{x \rightarrow a} c = c \quad \text{dimana } c \in \text{Real}$$

$$(2) \lim_{x \rightarrow a} c \cdot f(x) = c \cdot \lim_{x \rightarrow a} f(x)$$

$$(3) \lim_{x \rightarrow a} [f(x) \pm g(x)] = \lim_{x \rightarrow a} f(x) \pm \lim_{x \rightarrow a} g(x)$$

$$(4) \lim_{x \rightarrow a} f(x) \cdot g(x) = \lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} g(x)$$

$$(5) \lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)}$$

$$(6) \lim_{x \rightarrow a} [f(x)]^n = [\lim_{x \rightarrow a} f(x)]^n$$

Untuk lebih jelasnya pemakaian teorema di atas dalam soal limit, ikutilah contoh soal berikut ini :

01. Jika diketahui  $\lim_{x \rightarrow 2} f(x) = 5$  dan  $\lim_{x \rightarrow 2} g(x) = -4$  maka dengan teorema limit hitunglah nilai :

$$(a) \lim_{x \rightarrow 2} [2f(x) + 3g(x)]^2$$

$$(b) \lim_{x \rightarrow 2} [g^2(x) - 6]^3$$

Jawab

$$\begin{aligned} (a) \lim_{x \rightarrow 2} [2f(x) + 3g(x)]^2 &= [2 \{ \lim_{x \rightarrow 2} f(x) \} + 3 \{ \lim_{x \rightarrow 2} g(x) \}]^2 \\ &= [2 \{5\} + 3 \{-4\}]^2 \\ &= [10 - 12]^2 \\ &= [-2]^2 \\ &= 4 \end{aligned}$$

$$\begin{aligned}
\text{(b) } \lim_{x \rightarrow 2} [g^2(x) - 6]^3 &= [ \{ \lim_{x \rightarrow 2} g(x) \}^2 - 6 ]^3 \\
&= [ \{-4\}^2 - 6 ]^3 \\
&= [ 16 - 6 ]^3 \\
&= [ 10 ]^3 \\
&= 1000
\end{aligned}$$

02. Dengan teorema limit hitunglah :

$$\text{(a) } \lim_{x \rightarrow 2} \left[ \frac{x^2 + 2x - 8}{x^2 - 6x + 8} \right]^2 \left[ \frac{x^2 + 6x - 16}{x^2 + x - 6} \right]^3$$

$$\text{(b) } \lim_{x \rightarrow 3} \left[ \frac{x^2 + 3x - 18}{x^2 - 7x + 12} \right] \sqrt{\frac{x^2 + 2x - 15}{x^2 - 5x + 6}}$$

Jawab

$$\begin{aligned}
\text{(a) } \lim_{x \rightarrow 2} \left[ \frac{x^2 + 2x - 8}{x^2 - 6x + 8} \right]^2 \left[ \frac{x^2 + 6x - 16}{x^2 + x - 6} \right]^3 \\
&= \left[ \lim_{x \rightarrow 2} \frac{x^2 + 2x - 8}{x^2 - 6x + 8} \right]^2 \left[ \lim_{x \rightarrow 2} \frac{x^2 + 6x - 16}{x^2 + x - 6} \right]^3 \\
&= \left[ \lim_{x \rightarrow 2} \frac{(x + 4)(x - 2)}{(x - 4)(x - 2)} \right]^2 \left[ \lim_{x \rightarrow 2} \frac{(x + 8)(x - 2)}{(x + 3)(x - 2)} \right]^3 \\
&= \left[ \lim_{x \rightarrow 2} \frac{(x + 4)}{(x - 4)} \right]^2 \left[ \lim_{x \rightarrow 2} \frac{(x + 8)}{(x + 3)} \right]^3 \\
&= \left[ \frac{2 + 4}{2 - 4} \right]^2 \left[ \frac{2 + 8}{2 + 3} \right]^3 \\
&= [3]^2 [2]^3 \\
&= 72
\end{aligned}$$

$$\begin{aligned}
\text{(b) } \lim_{x \rightarrow 3} \left[ \frac{x^2 + 3x - 18}{x^2 - 7x + 12} \right] \sqrt{\frac{x^2 + 2x - 15}{x^2 - 5x + 6}} \\
&= \left[ \lim_{x \rightarrow 3} \frac{x^2 + 3x - 18}{x^2 - 7x + 12} \right] \sqrt{\lim_{x \rightarrow 3} \frac{x^2 + 2x - 15}{x^2 - 5x + 6}} \\
&= \left[ \lim_{x \rightarrow 3} \frac{(x + 6)(x - 3)}{(x - 4)(x - 3)} \right] \sqrt{\lim_{x \rightarrow 3} \frac{(x + 5)(x - 3)}{(x - 2)(x - 3)}}
\end{aligned}$$

$$\begin{aligned}
&= \left[ \lim_{x \rightarrow 3} \frac{(x+6)}{(x-4)} \right] \sqrt{\lim_{x \rightarrow 3} \frac{(x+5)}{(x-2)}} \\
&= \left[ \frac{3+6}{3-4} \right] \sqrt{\frac{3+5}{3-2}} \\
&= -9\sqrt{8} \\
&= -9(2\sqrt{2}) \\
&= -18\sqrt{2}
\end{aligned}$$

03. Dengan teorema limit hitunglah :

$$\begin{aligned}
\text{(a)} \quad &\lim_{x \rightarrow \infty} \left[ \frac{4x^3 - 2x + 5x^2}{6x - 2x^3} \right]^3 \sqrt{\frac{3x - 12x^2 + 5}{6 + 4x - 3x^2}} \\
\text{(b)} \quad &\lim_{x \rightarrow \infty} \left[ \frac{6x^2 - 8x^4 - 4}{4x^4 - 3x} \right]^3 \left[ \frac{x^3 - 5x + 2}{x^2 - 6x - 4x^3} \right]^2
\end{aligned}$$

Jawab

$$\begin{aligned}
\text{(a)} \quad &\lim_{x \rightarrow \infty} \left[ \frac{4x^3 - 2x + 5x^2}{6x - 2x^3} \right]^3 \sqrt{\frac{3x - 12x^2 + 5}{6 + 4x - 3x^2}} \\
&= \left[ \lim_{x \rightarrow \infty} \frac{4x^3 - 2x + 5x^2}{6x - 2x^3} \right]^3 \sqrt{\lim_{x \rightarrow \infty} \frac{3x - 12x^2 + 5}{6 + 4x - 3x^2}} \\
&= \left[ \frac{4 - 0 + 0}{0 - 2} \right]^3 \sqrt{\frac{0 - 12 + 0}{0 + 0 - 3}} \\
&= \left[ \frac{4}{-2} \right]^3 \sqrt{\frac{-12}{-3}} \\
&= (-2)^3 \sqrt{4} \\
&= -16
\end{aligned}$$

$$\begin{aligned}
\text{(b)} \quad &\lim_{x \rightarrow \infty} \left[ \frac{6x^2 - 8x^4 - 4}{4x^4 - 3x} \right]^3 \left[ \frac{x^3 - 5x + 2}{x^2 - 6x - 4x^3} \right]^2 \\
&= \left[ \lim_{x \rightarrow \infty} \frac{6x^2 - 8x^4 - 4}{4x^4 - 3x} \right]^3 \left[ \lim_{x \rightarrow \infty} \frac{x^3 - 5x + 2}{x^2 - 6x - 4x^3} \right]^2
\end{aligned}$$

$$\begin{aligned} &= \left[ \frac{0-8-0}{4-0} \right]^3 \left[ \frac{1-0+0}{0-0-4} \right]^2 \\ &= \left[ \frac{-8}{4} \right]^3 \left[ \frac{1}{-4} \right]^2 \\ &= -1/2 \end{aligned}$$